MATH858T: Stochastic methods with applications
Spring 2023

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The goal of this course is to give an introduction to stochastic methods used for the analysis of complex physical, chemical, and biological systems, and explain mathematical foundations of these methods. The focus is on examples, calculations and applications.

References: I will provide lecture notes for each topic. Besides, links to additional online teaching materials or references will be provided for each topic, if available.
Homework: there will be 5-6 homework problems assigned throughout the semester
Final: take-home
Grades: HW: 30%, in-class midterm 30%, take-home final: 40%

Tentative Outline of topics

Basic concepts of Probability
— Random Variables, Distributions, and Densities
— Expected Values and Moments
— The Law of Large Numbers
— The Central Limit Theorem
— Conditional Probability and Conditional Expectation

Maximum likelihood estimation
— Large sample properties
— The Delta method
— Asymptotic efficiency

Monte Carlo
— Sampling method
— Variance reduction: importance sampling, control variate, conditional MC
— Kernel density estimation

Markov Chains
— Discrete-time Markov Chains
— Continuous-time Markov Chains
— Markov Chain Monte Carlo Algorithms (Metropolis and Metropolis-Hastings)
— Transition Path Theory and Path Sampling Techniques
— Metastability and Spectral Theory
— Bayesian filtering

Brownian Motion

— Definition of Brownian Motion
— Brownian Motion and Heat Equation
— An Introduction to Stochastic Differential Equations (SDEs)
— Numerical integration of Stochastic ODEs: Euler-Maruyama, Milstein's, MALA
— Basics of stochastic optimal control

An Introduction into the Large Deviation Theory

— The Freidlin-Wentzell Action Functional
— Maximum likelihood paths
— Methods for computing Maximum likelihood paths and saddle points